mined experimentally. The scatter gives a measure of the accuracy of the photoelastic technique in determining strain and also indicates the variation of stress in the shell. The usual procedure is to plot stress ratio (σ/σ_{cr}) vs end deformation of the shell. However, the author simply wanted to illustrate a typical test run using the photoelastic method of computing strain.

Recent results from several photoelastic shells tested by the author have confirmed the fact that buckling loads within 10% of the classical value are obtainable. Although the shells employed had relatively low ratios of radius to thickness (i.e., $100 \leq R/t \leq 170$), the buckling loads represent an increase (about 10%) over the data in Fig. 1 of the preceding comment for the same value of R/t. The author admits that the results of Ref. 2 represent a "significant improvement over the general body of existing data" because of the higher R/t ratio of the shells tested.

In the concluding remarks of Ref. 1, the author wished to point out the following:

1) Of the two existing competitive theories explaining the lower buckling loads (well below the classical value), Tsien's energy criterion, as well as having no logical basis,[3] appeared to have no experimental basis. Tsien's lower buckling load, as stated by the author,[1] was achieved only by applying a lateral disturbance to the shell.

2) When isotropic elastic shells are made sufficiently free of imperfections and loaded axially in a rigid test machine, buckling loads very close to the classical value can be achieved. The author never intended his note to constitute unique evidence of the importance of imperfections on the buckling load of a shell loaded axially in compression. Furthermore, his *tentative* conclusion was proposed knowing full well that further investigation of both end effects and imperfections on the shell buckling phenomena was necessary.

The author recognizes the fact that the classically predicted critical buckling load of a circular cylindrical shell can be attained only within certain limits. He also accepts the fact that classical theoretical solutions have assumed shell edges that are free to expand radially, and, since his cylinder edges were clamped, this posed a limit on the observed buckling loads. The theoretical analysis of Ref. 4 for the case of simply supported ends clearly indicates that the end conditions do play a role in reducing the critical buckling stress of a shell. However, end constraints did not appear to produce a serious effect on the clamped shells used by the author. Buckling loads were repeatable and always near 10% of the computed value. No appreciable end effect has been observed by the author in the photoelastic analysis of the shells, either in the prebuckled state or in the buckled configuration. Any bending deformations should appear as color striations at the ends of the loaded shell.

In conclusion, the author agrees with Leonard that very convincing experimental evidence of the effect of imperfections on the lowering of buckling loads is offered by the results of Ref. 2. However, it must be noted that the works of both Refs. 2 and 4 did not appear until December 1962, after the author had submitted his note (August 1962) for publication.

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Comment on "Flight Mechanics of the 24-Hour Satellite"

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In his article on the flight mechanics of the 24-hr satellite, Perkins[1] refers to an unpublished manuscript by Blitzer, Boughton, Kang, and Page[2] as having "shown that the rate of longitudinal drift of the 24-hr equatorial satellite due to equatorial oblateness is sufficiently large to be of concern to system designers." He further states that "With the exception of the period for the case of small amplitude oscillation, Blitzer's work did not present any closed-form analytical expressions for the mean path motion of the satellite."

The object of this note is to update Perkins' reference and inform the reader of two papers by Blitzer et al.[3, 4] which appeared prior to Perkins' article and in which the restrictions to equatorial orbits and small-amplitude longitudinal oscillations were removed. In the latter paper,[4] particularly, closed-form analytic expressions are given for the period and for both radial and longitudinal motions. Diurnal and long-period components are included in the expressions. The solution for the mean motion was motivated by noting the similarity of the problem to that of the physical pendulum.

Perkins adopts an independent approach to the problem, but our results appear to be consistent. The reader will also find some other recent papers[5, 6] to be of relevance.

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Comment on "Dynamic Analysis for Lunar Alightment"

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In a recent paper by Cappelli,[1] an analytical procedure was described for obtaining the motion during touchdown of a spacecraft landing on the lunar surface. Comparison of some

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results of Ref. 1 with a touchdown dynamics program that is in use at NASA Marshall Space Flight Center indicates that several assumptions made in Ref. 1 are questionable and lead to optimistic results.

One assumption made in the analysis is that the coefficient of friction is considered to be sufficiently high, for all landing conditions, to develop the external forces required to crush the energy-absorbing materials. This assumption is valid only as long as sliding is present. When a foot stops sliding, the force component acting along the surface is greatly reduced, and the lateral energy absorber stops crushing. In the example presented, the crushing force of the lateral energy absorber was taken to be half that of the vertical energy absorber. As the vehicle contacts a 15° slope on the uphill leg with the lateral velocity in the downhill direction (Fig. 5, p. 1122, Ref. 1), the force vector initially acting on the foot pad is inclined 26.6° from the vertical and thus 41.6° from the normal to the surface. Thus, a coefficient of friction of 0.89 or larger would be sufficiently high to develop the external force required initially to crush the lateral energy absorber. However, this condition exists for only a very short time. The foot stops sliding, and the component of force acting laterally is less than the crushing force and must be computed. At this point, the vehicle's leg is still crushing vertically but not laterally.

As the vehicle continues to rotate, the crushing rate of the vertical energy absorber reduces. At some point, the crushing rate changes sign. When this occurs, the time at which the vertical crushing rate goes to zero should be determined by interpolation. The vehicle may then rotate as a rigid body without any crushing. The vertical component of external force is then less than the crushing force of the vertical energy absorber and must be computed. Thus another assumption in the analysis of Ref. 1 is not valid, inasmuch as it is assumed that the vehicle's leg has lifted off the surface whenever the crushing rate has changed sign or, as stated in Ref. 1, whenever the amount of deformation decreases in magnitude. According to this assumption, after the downhill legs contact the surface and the amount of deformation decreases, the downhill legs would be considered to have lifted off the surface! This is incorrect, inasmuch as inelastic energy-absorption devices without rebound are considered. Actually, the crushing stops, and the vehicle then rotates on the downhill legs as a rigid body. It thus appears that the analysis of Ref. 1 fails to determine adequately the lateral and vertical forces whenever they are less than the respective crushing forces of the energy absorbers.

A comparison of the results from Fig. 5 of Ref. 1 with that obtained by the touchdown dynamics program in use at NASA Marshall Space Flight Center is shown in Fig. 1. The nomenclature of Ref. 1 is used. Exact duplication of the input data was not possible, since a lateral energy absorber crushing force is assumed in Ref. 1, whereas the program at NASA Marshall Space Flight Center assumes a value for the coefficient of friction. However, as the vehicle contacts on the two downhill legs, the legs are essentially normal to the surface so that the tangential force developed is half the normal force and corresponds to a friction coefficient of 0.5. Therefore, this value was used in the NASA Marshall Space Flight Center program to represent most accurately the lateral energy absorber.

As seen from Fig. 1, with a lateral velocity of zero, the vehicle tumbles by the procedure used at NASA Marshall Space Flight Center for a vertical velocity of about 1.9 m/sec or greater, whereas by the method of Ref. 1 the vehicle does not tumble until the vertical velocity exceeds about 3.1 m/sec. This represents an increase of over 60% in the permissible vertical velocity.

Up to a vertical velocity of about 4.3 m/sec, the uphill leg stops crushing but does not leave the surface. The vehicle rotates as a rigid body, and the vehicle's angular acceleration continues to increase the verticle's rotational rate in the tumbling sense. At a vertical velocity slightly in excess of 4.3 m/sec, the uphill leg stops crushing and does leave the surface. The computed normal force required to hold the leg to the surface becomes negative. No negative normal forces are physically possible, so that the forces are set to zero and the leg leaves the surface. When in free flight, the angular acceleration is zero, and thus the rotational rate does not increase. This accounts for the increased stability effect seen in Fig. 1. The discontinuity on the stability curve occurs at the vertical velocity which results in a free-flight condition after the uphill leg stops crushing.

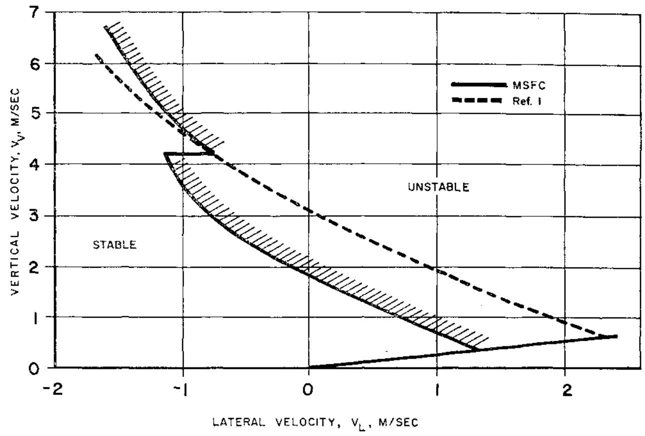

Fig. 1 Landing stability of vehicle impacting a 15° slope initially on one leg (R_1 = 12 ft, R_2 = 10.4 ft, R_3 = 6 ft, I_x/m = 36 ft², DCG = 10 ft, nF_v/m = 386.4 ft/sec², A = 15°, B = 0°, β = 270°).

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Addendum to "Nonequatorial Launching to Equatorial Orbits and General Nonplanar Launching"

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Attention is invited to the paper by Carstens and Edelbaum,[1] which gives data for optimum attainment of circular orbits from arbitrary launch points, considers nonapogee target orbit interception, and bears directly on the problem treated in Ref. 2. This reference was inadvertently and unfortunately omitted from the list of references given in Ref. 2.

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